

Parameterization of continua caused by gaseous constituents

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Abstract: The calculation of broadband continua caused by atmospheric trace gases like O₂, N₂, H₂O, CO₂ is described. The collision induced bands of O₂ and N₂, due to interaction of molecule pairs, are simulated by empirical models from literature. For H₂O the well know CKD continuum is used where a special treatment of the line wing calculation is necessary. For CO₂ an own continuum was determined, taking into account contributions more than 25cm⁻¹ apart from the line-center.

1 Introduction

The emission of the terrestrial atmosphere in the infrared spectral region is dominated by vibrational-rotational bands of a large number of atmospheric trace species such as H₂O, CO₂, O₃, CH₄, N₂O. In the case of measuring their emission with high spectral resolution the individual vibrational-rotational lines of most trace species can be resolved. In addition to the molecular lines, absorption features extending over wide spectral regions and varying slowly with frequency are present in atmospheric infrared emission spectra. This so-called *continuum* radiance has its origin in different physical mechanisms:

- collision-induced absorption bands due to the interaction of molecule pairs like N₂-N₂, N₂-O₂, O₂-O₂,
- superposition of line wings from strongly absorbing bands such as the 6.3 μm ν₂ band of H₂O and the 4.3 μm ν₃ band of CO₂

The different gaseous continua are calculated in the module **gascon_m** which include calls of the individual subroutines for the different continua.

2 O₂ Continuum

The O₂ Continuum is calculated by the subroutine `calc_o2cont@gascon_m`. The implementation is based on the empirical model by [1]. They combined laboratory measurements of normalized binary absorption coefficients $B_{O_2-O_2}$ and $B_{N_2-O_2}$ to the normalized absorption coefficient B_{O_2-air} assuming a composition of 21% O₂ and 79% N₂. The temperature and wavenumber dependence of the normalized absorption coefficient for O₂-air is modeled using a simple empirical law:

$$B_{O_2-air}(\nu, T) = B_{O_2-air}^0(\nu) \exp \left[\beta_{O_2-air}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (1)$$

with the reference temperature $T_0 = 296$ K and the normalized absorption coefficient $B_{O_2-air}^0$ and an empirically determined exponent for the temperature dependence $\beta_{O_2-air}^0$ at reference temperature, both tabulated as a function of wavenumber in the wavenumber region 1365 cm^{-1} to 1800 cm^{-1} in steps of 5 cm^{-1} . The normalized absorption coefficient $B_{O_2-air}^0$ and the exponent for the temperature dependence $\beta_{O_2-air}^0$ are given in units of [$\text{cm}^{-1} \text{ Am}^{-2}$] and [K], respectively. For calculation of the O₂ continuum at arbitrary wavenumbers these coefficients are linearly interpolated. The absorption cross-section σ_{O_2} of the O₂ continuum in units of [cm^2/molec] can then be calculated by:

$$\sigma_{O_2}(\nu, \rho_{O_2}, T) = \frac{\rho_{O_2} \times B_{O_2-air}^0(\nu)}{(0.21 \text{ Am})^2} \times \exp \left[\beta_{O_2-air}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (2)$$

where ρ_{O_2} is the density of O₂ in units of [molec/cm^3].

The subroutine `calc_o2cont@gascon_m` also supports the calculation of the derivative of the O₂ absorption cross section with respect to temperature given by:

$$\begin{aligned} \frac{d\sigma_{O_2}}{dT}(\nu, \rho_{O_2}, T) &= \frac{\rho_{O_2} \times B_{O_2-air}^0(\nu)}{(0.21 \text{ Am})^2} \times \frac{\beta_{O_2-air}^0(\nu)/T - 1}{T} \\ &\times \exp \left[\beta_{O_2-air}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \end{aligned} \quad (3)$$

3 N₂ Continuum

The N₂ Continuum is calculated by the subroutine `calc_n2cont@gascon_m`. The implementation is based on the empirical model by [2]. The temperature and wavenumber dependence of the normalized absorption coefficient $B_{N_2-N_2}$ for pure N₂ is modeled using a simple empirical law:

$$B_{N_2-N_2}(\nu, T) = B_{N_2-N_2}^0(\nu) \exp \left[\beta_{N_2-N_2}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (4)$$

with the reference temperature $T_0 = 296$ K and the normalized absorption coefficient $B_{N_2-N_2}^0$ and an empirically determined exponent for the temperature dependence $\beta_{N_2-N_2}^0$ at reference temperature, both tabulated as a function of wavenumber in the wavenumber region 2125 cm^{-1} to 2600 cm^{-1} in steps of 5 cm^{-1} . The normalized absorption coefficient $B_{N_2-N_2}^0$ and the exponent for the temperature dependence $\beta_{N_2-N_2}^0$ are given in units of [$\text{cm}^{-1} \text{ Am}^{-2}$] and [K], respectively. For calculation of the N₂ continuum at arbitrary wavenumbers these coefficients are linearly interpolated. The relative efficiency $E_{O_2/N_2}^{N_2}$ of collisions of N₂ with O₂ and N₂ molecules can be modeled by the following wavenumber-independent expression:

$$E_{O_2/N_2}^{N_2}(T) = 1.294 - 0.4545(T/T_0) \quad (5)$$

The N₂-air collision-induced spectra in the Earth's atmosphere from the pure N₂ data can then be calculated by:

$$B_{N_2-air}(\nu, T) = [0.79 + 0.21 \times E_{O_2/N_2}^{N_2}(T)] \times B_{N_2-N_2}(\nu, T) \quad (6)$$

The absorption cross-section σ_{N_2} of the N₂ continuum in units of [cm²/molec] considering the composition of air by 79% N₂ and 21% O₂ can then be calculated by:

$$\begin{aligned} \sigma_{N_2}(\nu, \rho_{N_2}, T) &= \frac{\rho_{N_2} \times B_{N_2-N_2}^0(\nu)}{(0.79 \text{ Am})^2} \times [0.8387 - 0.0754 \frac{T}{T_0}] \\ &\times \exp \left[\beta_{N_2-N_2}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \end{aligned} \quad (7)$$

where ρ_{N_2} is the density of N₂ in units of [molec/cm³].

The subroutine **calc_n2cont@gascon_m** also supports the calculation of the derivative of the N₂ absorption cross section with respect to temperature given by:

$$\begin{aligned} \frac{d\sigma_{N_2}}{dT}(\nu, \rho_{N_2}, T) &= \frac{\rho_{N_2} \times B_{N_2-N_2}^0(\nu)}{0.79 \text{ Am}^2} \times \exp \left[\beta_{N_2-N_2}^0(\nu) \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \\ &\times \left[\frac{0.8387 - 0.0754 \frac{T}{T_0}}{T} \left(\frac{\beta_{N_2-N_2}^0(\nu)}{T} - 1 \right) - \frac{0.0754}{T_0} \right] \end{aligned} \quad (8)$$

4 H₂O Continuum

The H₂O Continuum is calculated by the subroutine **calc_h2ocont@gascon_m**. The implementation is based on the CKDv2.2 model by [3] This model introduces an empirical lineshape correction factor (χ -factor) which accounts for the non-Lorentzian behaviour of H₂O lines. The specific realization of Clough's continuum parameterization is coupled with the definition of the continuum: The absorption of all water vapor transitions is separated into the *local absorption* and the *continuum absorption*. The local absorption is defined as a Lorentzian lineshape out to $\pm 25 \text{ cm}^{-1}$ apart from the line center, minus the Lorentz value at 25 cm^{-1} . The continuum is then simply defined by any observed absorption not attributable to the local absorption.

Application of this definition of the H₂O continuum to KOPRA calculations requires the subtraction of the so-called *basement*, which is the Lorentz value at 25 cm^{-1} , from the absorption coefficient of each individual H₂O line out to $\pm 25 \text{ cm}^{-1}$ apart from the line center. Furthermore, calculation of absorption coefficient of H₂O lines is restricted to the region inside of $\pm 25 \text{ cm}^{-1}$ apart from the line center. Therefore, a switch is introduced into the subroutine **add_lines_chilm@addlin_m** to perform this restriction and subtraction for H₂O lines in the case the H₂O continuum is included.

The original CKD model was developed by fitting the parameters in analytical χ -factors to reach agreement between the calculated continuum and laboratory measurements. For pragmatic reasons these χ -factors were used to calculate continuum coefficients $C_s^0(\nu, T)$ for the self-broadening component and $C_f^0(\nu)$ for the foreign-broadening component at reference pressure p_{ref} such that the continuum absorption cross section in units of [cm²/molec] is given by:

$$\begin{aligned} \sigma_c(\nu, T, vmr_{H_2O}) &= \nu \tanh \left(\frac{hc\nu}{2kT} \right) \left(\frac{p}{p_{ref}} \right) \left(\frac{T_{ref}}{T} \right) \\ &\times [vmr_{H_2O} C_s^0(\nu, T) + (1 - vmr_{H_2O}) C_f^0(\nu)] \end{aligned} \quad (9)$$

Values of C_s^0 are tabulated for temperatures of 296 K and 260 K and C_f^0 for a temperature of 296 K in the wavenumber range from -20 cm^{-1} up to 20000 cm^{-1} in steps of 10 cm^{-1} . For calculation of the water vapor continuum at arbitrary wavenumbers these coefficients are interpolated linearly in wavenumbers. In the case of C_s^0 an exponential interpolation to arbitrary temperatures is performed.

The present CKDv2.2 model includes modifications of the original CKDv0 model to maintain better agreement with laboratory and atmospheric continuum measurements. These modifications are performed by correction factors to the continuum coefficients to reduce deviations with measurements in specific spectral regions.

Derivatives of the water vapor continuum with respect to temperatures are implemented by calculating the differential quotient for a temperature increment of $dT = 0.1 \text{ K}$.

5 CO₂ Continuum

The CO₂ continuum absorption arises from the superposition of the extreme wings of many distant lines. To account for this CO₂ continuum in accurate line-by-line calculations it would be required to include CO₂ lines over a very large spectral region. This method would increase the required computation time dramatically. Therefore, CO₂ continuum coefficients are precalculated for different temperatures as a function of wavenumber.

The generation of CO₂ continuum coefficients is performed by a stand-alone program `create_co2coe.f90`. This program calculates the CO₂ absorption cross section at user-defined spectral positions and user-defined temperatures by superimposing all spectral lines out of a spectral region which also can be specified by the user. The spectral line shape used for the calculation of the continuum coefficients is the Lorentzian line shape modified by so-called χ -factors which account for the non-Lorentzian behavior of CO₂ lines. We used the asymmetric χ -factors of Menoux et al. (1987, 1991). As the temperature dependence of the asymmetric χ -factors for the O₂ broadening has not been investigated and, on the other hand, the temperature dependence of the symmetric χ -factors indicate significant differences for N₂- and O₂ broadening we applied for O₂ broadening at low temperatures symmetric χ -factors.[4] These χ -factors are interpolated and extrapolated linearly in temperature. In cases where extrapolation leads to negative χ -factors they are set to zero.

The continuum cross-section includes all contributions of line wings out of $\pm 25 \text{ cm}^{-1}$ apart from the line center plus the χ -modified Lorentz value at 25 cm^{-1} of each CO₂ line. Application of these continuum cross-section to KOPRA line-by-line calculations requires the subtraction of the χ -modified Lorentz value at 25 cm^{-1} from the absorption coefficient of each individual CO₂ line out to $\pm 25 \text{ cm}^{-1}$. Furthermore, calculation of absorption coefficient of CO₂ lines is restricted to the region inside of $\pm 25 \text{ cm}^{-1}$ apart from the line center. Therefore, a switch is introduced into the subroutine `add_lines@addlin_m` to perform this restriction and subtraction for CO₂ lines in the case the CO₂ continuum is included.

The CO₂ continuum coefficients to be used by KOPRA are stored in the module `co2coe.m`. They were calculated for 6 different temperatures covering the range from 150 K to 350 K and for wavenumbers from 0 to 10000 cm^{-1} with 5 cm^{-1} spacing. The continuum coefficients can be assumed to be linear in pressure because the Lorentz line shape is only applied to regions more than 25 cm^{-1} apart from the line center where the air broadening half width is negligible to the distance from the line center. Therefore, the continuum coefficients stored in `co2coe.m` have units of $[\frac{\text{cm}^2}{\text{molec hPa}}]$.

In the subroutine **calc_co2cont@gascon_m** these continuum coefficients are interpolated linearly to the actual wavenumber and interpolated by the Steffen Interpolation scheme (subroutine **intp_steffen@varsub_m**) to the actual temperature. In order to obtain the continuum absorption cross-sections in units of [$\frac{cm^2}{molec}$] the interpolated coefficients are multiplied by the actual pressure. Derivatives of the CO₂ continuum with respect to temperatures are implemented by calculating the differential quotient for a temperature increment of $dT = 0.1$ K.

Bibliography

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